

General Certificate of Education  
June 2008  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 2**

**MPC2**

Thursday 15 May 2008 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Write  $\sqrt{x^3}$  in the form  $x^k$ , where  $k$  is a fraction. (1 mark)

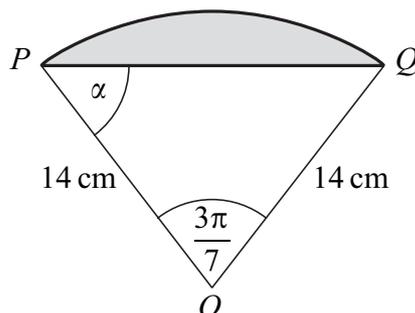
(b) A curve, defined for  $x \geq 0$ , has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find  $\frac{dy}{dx}$ . (3 marks)

(ii) Find the equation of the tangent to the curve at the point where  $x = 4$ , giving your answer in the form  $y = mx + c$ . (5 marks)

2 The diagram shows a shaded segment of a circle with centre  $O$  and radius 14 cm, where  $PQ$  is a chord of the circle.



In triangle  $OPQ$ , angle  $POQ = \frac{3\pi}{7}$  radians and angle  $OPQ = \alpha$  radians.

(a) Find the length of the arc  $PQ$ , giving your answer as a multiple of  $\pi$ . (2 marks)

(b) Find  $\alpha$  in terms of  $\pi$ . (2 marks)

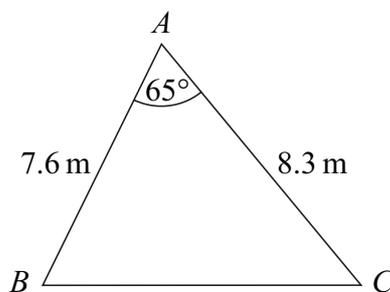
(c) Find the **perimeter** of the shaded segment, giving your answer to three significant figures. (2 marks)

3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \dots$$

- (a) Find the common ratio of the series. (1 mark)
- (b) Find the sum to infinity of the series. (2 marks)
- (c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. (2 marks)
- (d) Prove that the  $n$ th term of the series is  $25 \times 0.8^n$ . (2 marks)

4 The diagram shows a triangle  $ABC$ .



The size of angle  $BAC$  is  $65^\circ$ , and the lengths of  $AB$  and  $AC$  are 7.6 m and 8.3 m respectively.

- (a) Show that the length of  $BC$  is 8.56 m, correct to three significant figures. (3 marks)
- (b) Calculate the area of triangle  $ABC$ , giving your answer in  $\text{m}^2$  to three significant figures. (2 marks)
- (c) The perpendicular from  $A$  to  $BC$  meets  $BC$  at the point  $D$ .

Calculate the length of  $AD$ , giving your answer to the nearest 0.1 m. (3 marks)

5 (a) Write down the value of:

- (i)  $\log_a 1$ ; (1 mark)
- (ii)  $\log_a a$ . (1 mark)

- (b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

find the value of  $x$ . (3 marks)

Turn over ►

6 The  $n$ th term of a sequence is  $u_n$ .

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where  $p$  and  $q$  are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \quad u_2 = 8 \quad u_3 = 4$$

- (a) Show that  $q = 6$  and find the value of  $p$ . (5 marks)
- (b) Find the value of  $u_4$ . (1 mark)
- (c) The limit of  $u_n$  as  $n$  tends to infinity is  $L$ .
- (i) Write down an equation for  $L$ . (1 mark)
- (ii) Hence find the value of  $L$ . (2 marks)

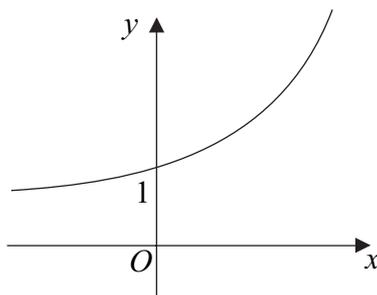
7 (a) The expression  $\left(1 + \frac{4}{x^2}\right)^3$  can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers  $p$  and  $q$ . (3 marks)

- (b) (i) Hence find  $\int \left(1 + \frac{4}{x^2}\right)^3 dx$ . (4 marks)
- (ii) Hence find the value of  $\int_1^2 \left(1 + \frac{4}{x^2}\right)^3 dx$ . (2 marks)

- 8 The diagram shows a sketch of the curve with equation  $y = 6^x$ .



- (a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for  $\int_0^2 6^x dx$ , giving your answer to three significant figures. (4 marks)
- (ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of  $\int_0^2 6^x dx$ . (2 marks)
- (b) (i) Describe a single geometrical transformation that maps the graph of  $y = 6^x$  onto the graph of  $y = 6^{3x}$ . (2 marks)
- (ii) The line  $y = 84$  intersects the curve  $y = 6^{3x}$  at the point  $A$ . By using logarithms, find the  $x$ -coordinate of  $A$ , giving your answer to three decimal places. (4 marks)
- (c) The graph of  $y = 6^x$  is translated by  $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$  to give the graph of the curve with equation  $y = f(x)$ . Write down an expression for  $f(x)$ . (2 marks)
- 9 (a) Solve the equation  $\sin 2x = \sin 48^\circ$ , giving the values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ . (4 marks)
- (b) Solve the equation  $2 \sin \theta - 3 \cos \theta = 0$  in the interval  $0^\circ \leq \theta < 360^\circ$ , giving your answers to the nearest  $0.1^\circ$ . (4 marks)

**END OF QUESTIONS**

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